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Modified Schema-Based Instruction to Encourage Mathematical Practice Use for a Student with Autism Spectrum Disorder

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Abstract: Mathematics education highlights the importance of helping students understand mathematical content and the ability to think critically about mathematics. Mathematical practice standards outline expectations for engaging students in meaningful mathematics activities to encourage a deep understanding of mathematical concepts and content. For some students, however, engagement in mathematical practices may not come naturally, and therefore will need to be explicitly taught. Students with autism spectrum disorder have communication and executive functioning deficits that are likely to decrease their engagement in mathematical practices. A multiple probe across behaviors single-case design was used to investigate the effectiveness of a modified schema-based instructional (MSBI) strategy on the use of mathematical practices of a middle school student with autism spectrum disorder (ASD). The student was taught to solve equal group, proportional, and percent of change word problems using MSBI. Results provide evidence of a functional relation between MSBI and mathematical practices with three demonstrations of an effect. Results from social validity questionnaires also support the importance of the skills taught, along with an observed behavioral change for the student during mathematics instruction in his classroom setting. Implications for teaching and future research are discussed.

Mathematics education is no longer thought to be a one-dimensional task requiring memorization of facts and figures. Students today are asked to use critical thinking to prepare them for unique situations they will face outside school walls. Mathematics education reform emphasizes the importance of both what students should know about mathematics as well as how students engage in mathematical tasks (Kilpatrick et al., 2001). The Common Core State Standards for Mathematics include not only content standards, but also standards for mathematical practice. The practice standards (i.e., mathematical practices) are based on process standards published by the National Council for Teachers of Mathematics (NCTM, 2000) along with the strands of mathematical proficiency reported in the National Research Council's report "Adding It Up" ([NRC], Kilpatrick et al., 2001). Together, the content and practice standards in the Common Core State Standards emphasize the importance of helping students understand mathematics and communicate their mathematical understanding. Difficulty in communication is one of the diagnostic criteria for individuals with autism spectrum disorder (ASD). Therefore, teachers need ways to help students with ASD communicate their mathematical thinking in a way that promotes their mathematical proficiency.

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Mathematics for Students with ASD

Students with ASD are a heterogenous group of individuals who demonstrate a variety of unique learning strengths (e.g., routine, detailed thinking) and needs (e.g., executive functioning, communication, and metacognition). Deficits in executive functioning (e.g., the ability to plan and organize information), communication (e.g., explaining mathematical reasoning), and metacognition (e.g., self-monitoring progress and reasonableness) are

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barriers that may interfere in the development of mathematical proficiency. Therefore, students with ASD may need both educational and social supports to actively participate in meaningful academic conversations to extend their understanding of the content and help them to develop conversation skills needed in many work environments.

Although there is no comprehensive mathematical profile for students with ASD, several reviews and studies offer insights into possible mathematical strengths and weaknesses that individuals with ASD may possess. Some researchers suggest students with ASD are more likely to possess average to above-average mathematical abilities (e.g. Chiang & Lin, 2007), although other research groups report a greater likelihood of a mathematics disability than mathematical giftedness (e.g., Oswald et al., 2016). Some of this disparity may be a result of how mathematics achievement was measured, as mathematical skills may vary by task. When comparing the mathematical abilities of students with ASD, students may demonstrate more success with computational skills than problem solving or critical thinking skills (Jones et al., 2009; Oswald et al., 2016). In fact, in a review of the literature, Schaefer Whitby and Mancil (2009) found students with "high functioning autism" (i.e., ASD with average to above-average IQ) possessed specific weaknesses related to solving mathematical word problems (i.e., organization, attention, and complex processing skills). These findings support the hypothesis that students with ASD are likely to need additional supports during higher grades, as mathematics becomes more complex with a greater focus on applied problems (Oswald et al., 2016).

Modified Schema Based Instruction

One instructional strategy that has shown potential to help individuals with ASD improve mathematical problem solving skills is modified schema-based instruction (MSBI), a variation of schema-based instruction (SBI). SBI uses principles of explicit instruction to teach students to use specific problem solving strategies that are linked to categories or types of word problems (e.g., proportion, equal group, percent of change; Jitendra et al., 2009). Students learn to identify the structure of a problem, and then visually represent the mathematical relationship in a schema (diagram) aligned with the problem type (Jitendra et al., 2009). MSBI maintains core instructional strategies of traditional SBI (i.e., explicit strategy instruction, teacher think-alouds, and visual representations; Spooner et al., 2017), while incorporating additional evidence-based practices for students with ASD (i.e., task analysis, systematic prompting, visual supports).

MSBI has been used effectively to improve problem solving skills for students with ASD and co-morbid intellectual disability (ID). Students have demonstrated their ability to not only learn to solve a variety of problem types, including additive comparison (Root et al., 2017; Root et al., 2019), additive equal group (Root & Browder, 2019; Root, Henning, et al., 2018), and percent of change (Root, Cox, et al., 2018; Root et al., 2019), but also to discriminate between problem types. Across studies that taught more than one type of problem, the majority of participants demonstrated that without explicit discrimination training, there was over-generalization (Root, Cox, et al., 2018; Root et al., 2019; Root, Henning, et al., 2018). This pattern of responding was also reported by Browder et al. (2019) in their evaluation of the effectiveness of MSBI for students with moderate ID. Further, participants with ASD and comorbid ID demonstrated consistent difficulties with generalization when supports such as task analysis or graphic organizers were taken away (Root & Browder, 2019; Root, Cox, et al., 2018; Root, Henning, et al., 2018). So while MSBI literature suggests it is effective in improving problem solving skills of students with ASD and co-morbid ID, it is unknown whether individuals with ASD who do not have comorbid intellectual disability will have the same pattern of responding and therefore require the same intensity of instruction.

Less attention has been directed toward the effects of MSBI on the problem solving ability of students with ASD who have average to above-average IQ. Given the heterogeneity of individuals with ASD, several researchers have called for more research to include students

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with ASD who have average to above-average IQ (Gevarter et al., 2016; King et al., 2016) to investigate what level of supports are appropriate for each group of students. Cox and Root (2020) used an ABAB reversal design to investigate the effectiveness of MSBI with provided visual supports on acquisition and maintenance of mathematics content and practices for middle school students with ASD without comorbid ID. Two students learned to solve proportional word problems containing extraneous information multiple ways while showing their work and explaining their mathematical reasoning. The researchers specifically evaluated the effects of MSBI on mathematical problem solving flexibility and communication. During baseline one of the two students could solve some of the word problems when just provided a blank sheet of paper with the word problem printed at the top. However, his accuracy was inconsistent and he did not show any work or explain his reasoning despite correctly solving some problems. During the intervention (MSBI) phases when they were provided with MSBI and a worksheet with visual supports (e.g., graphic organizer with nine-step task analysis and schematic diagrams), both students used multiple strategies to correctly solve the problems and explained their mathematical reasoning. This indicated a meaningful change from their baseline performance, when they did not have access to the instruction or the visual supports provided during intervention. Performance of both participants returned to baseline levels when MSBI and corresponding visual supports were removed. Due to design limitations, it is not possible to know which components of the MSBI instructional package were needed for each student. Authors called for future research to investigate whether the graphic organizer without the additional explicit instruction provided during MSBI (i.e.; model, lead, test) would produce different results, and if the visual supports could be faded while maintaining independent problem solving. MSBI may be a viable strategy to improve student's mathematical proficiency, but replications of results across different types of word problems and participants are needed to expand generalizability.

Purpose

The aim of the current study was to extend previous research on the effectiveness of MSBI to improve mathematical problem solving skills for students with ASD without ID. Specifically, this investigation examined the effects of MSBI on the acquisition and generalization of mathematical practices of a student with ASD who did not have co-morbid ID. The research questions were: (1) Is there a functional relation between the use of modified schema-based instruction (MSBI) and the increased use of mathematical practices (as measured by a researcher created rubric) of a student with ASD when solving multiplicative word problems? (2) What are the effects of the use of MSBI and the participant's ability to complete multiplicative word problems with reduced visual supports?

Method

Participants

Approval from a university human subjects committee, parental consent, and student assent were obtained prior to onset of research activities. To be eligible to participate, the following inclusion criteria had to be met: (a) medical or educational diagnosis of ASD; (b) 10 to 13 years of age; (c) teacher recommendation based on perceived need for problem solving instruction, and (d) no educational or medical diagnosis of ID. Researchers met with personnel at a private school for students with ASD to describe the purpose of the research and recruit participants. Students who were below grade level in mathematical problem solving skills, despite adequate procedural skills were recruited. Teachers then sent home information and consent forms to eligible families. No standardized cognitive or behavioral assessment scores were available, but the private school was limited to students with an autism spectrum diagnosis.

Ron was a 12-year-old, White male enrolled full-time in the sixth grade at a private school for students with ASD. Ron received instruction throughout the day from a non-certified teacher in a small group setting. Through the use of a researcher designed pre-screening

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TABLE 1

Example Word Problems

Equal Group	*Proportion*	*Percent of Change*
Jordan is making birthday bags for his party. He is putting 8 candies in each bag. If he makes 20 bags for his party, how many candies does he need to buy?	The Green Arrow goes through 6 quivers in 2 days while fighting crime. How many quivers would he use in 8 days?	Tonia got her nails done at the nail salon before her wedding. Her manicure cost $20. She left a 25% tip. What will her total cost be?
Heather bought scrapbook paper from Michaels. She bought 5 packages of paper. If each package contains 20 pieces of paper, how many pieces of paper did she buy in all?	Tracey paints pottery for her cousin's store. In 4 hours, Tracey can paint 16 cups. If Tracey paints the same number of cups every hour, how many cups does Tracey paint in 8 hours?	Nick ordered pizza for his guys night. His total bill was $38. He wanted to leave a 15% tip. How much is his total bill?

assessment and the Test of Mathematical Abilities – Third Edition (TOMA-3; Brown et al., 2012), the researcher assessed the participant's computation (2nd percentile, Grade Equivalent = 3.4) and word problem skills (37th percentile, Grade Equivalent = 5.2) as below grade-level. Ron's score on the Everyday Mathematics Subsection of the TOMA-3, however, demonstrated a strength in his awareness of mathematics in everyday life (e.g., There are 52 cards in a deck; 63rd percentile, Grade Equivalent = 8.7). During the prescreening, the researcher observed that Ron was able to use the calculator to solve some simple additive mathematical word problems, but he did not show any work. When asked how he solved a problem or why he had solved it that way, Ron replied "I just knew". The report from the teacher and school director also confirmed these observations. Both reported Ron infrequently shared his mathematical thinking in either written or verbal form. Following the advice and procedures of the school, behavioral supports (i.e., token economy) were used throughout the study to encourage focus and task completion.

Setting and Interventionists

The research took place over three months, with sessions occurring three to four days per week for approximately 20 minutes. All sessions occurred during the regular school day in a separate classroom within the private school. Sessions were conducted in a one-on-one format. The first author (a doctoral candidate in special education and a former middle school mathematics teacher) served as the primary interventionist, while the second author (special education faculty member) and a trained undergraduate research student served as secondary interventionists as needed. Following procedures of the school, a behavioral therapist also sat in the room but did not interact with the student.

Materials

Three types of word problems were taught in this study; (a) equal group (EG), (b) proportional (PROP), and (c) percent of change (PC). Examples of each type of problem are shown in Table 1 Multiplicative problems were selected, as they are a crucial component of middle grades mathematics and formative to the development of algebraic understanding. In addition, the three problem types were distinct enough to control for carryover effects. The three targeted problem types aligned with middle grade Common Core State Standards; CCSS.MATH.CONTENT.7.EE.B.3 (solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form, using tools strategically), CCSS.MATH.CONTENT.7.RPA.2 (recognize and represent proportional relationships between quantities),

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and CCSS.MATH.CONTNET.7.RPA.3 (use proportional relationships to solve multistep ratio and percent problems).

Approximately 60 problems for each problem type were written by the first author based on recommendations from previous MSBI research (Spooner et al., 2017). A professor in the mathematics education department assessed problems for content validity and suggested changes were made prior to the study. To decrease testing threats, word problems were randomly assigned, and no problems were used more than once. During baseline, probe, and intervention sessions the word problems were displayed on top of a laminated sheet of computer paper containing the heuristic (see Figure 1). The schematic diagrams (i.e., graphic organizers; see Figure 1) were connected to the right-hand side of a legal-sized clipboard with Velcro, which allowed the participant to select them, move them onto the heuristic, and write on them with a dry erase marker. The schematic diagrams were created based on previous SBI and MSBI research (e.g., Cox & Root, 2020; Jitendra et al., 2009; Spooner et al, 2017). The student also had access to a dry erase marker, washcloth to erase, and a standard four-function calculator during all baseline, probe, and intervention sessions.

Materials for generalization sessions had reduced visual supports compared to those provided to the participant in baseline/probe/intervention sessions. During generalization sessions, the word problem was printed at the top of a sheet of computer paper that also contained a vertical four-step task analysis (see Figure 2), a pencil, and four-function calculator. The participant did not have access to the heuristic or schemas during generalization sessions.

Design and Measurement

A single case, multiple probe across behaviors research design (Ledford & Gast, 2018) was used to demonstrate a functional relation between MSBI and mathematical practices. Multiple probe designs use repeated measures of the dependent variable before and after the introduction of the intervention to compare performance during different conditions. Previous studies suggest individuals with ASD have difficulty generalizing mathematical problem solving skills across unknown problem types (e.g., Root, Cox, et al., 2018; Root, Henning, et al., 2018), therefore a multiple probe across behaviors design was used, as the behaviors (i.e., three problem types) were thought to be independent. The three problem types included equal group (EG), proportion (PROP) and percent of change (PC). A multiple probe design was selected rather than a multiple baseline to reduce threats to testing fatigue (Ledford & Gast, 2018).

There were four experimental conditions: (a) baseline, (b) intervention, (c) probe, and (d) generalization. Intervention (MSBI) was staggered across the three problem types to allow for three demonstrations of an effect. A minimum of five baseline data points were collected across all three problem types before intervention began on the first problem type (EG). Once the participant met mastery criteria of a minimum of 12/14 points over two sessions in EG, a series of three probes (using baseline conditions) allowed for analysis of mathematical practices across all problem types. This probe condition assessed performance without feedback on the recently taught problem type (EG), as well as generalization of practices to the two untaught problem types (PROP and PC). Following the three-session probe phase, generalization with reduced supports was assessed for each problem type. The participant then began intervention for the second problem type (PROP). Once he met mastery criteria, the series of three probes were again administered to assess performance without feedback on PROP, maintenance of mathematical practices for the previously mastered problem type (EG), and generalization of practices to the untaught problem type (PC). Following the generalization session, the participant began intervention in the final problem type (PC). After meeting mastery criteria, the series of three probes across all three problem types assessed maintenance without feedback. Next, discrimination sessions were conducted across the three problem types prior to a final series of three probes and generalization session.

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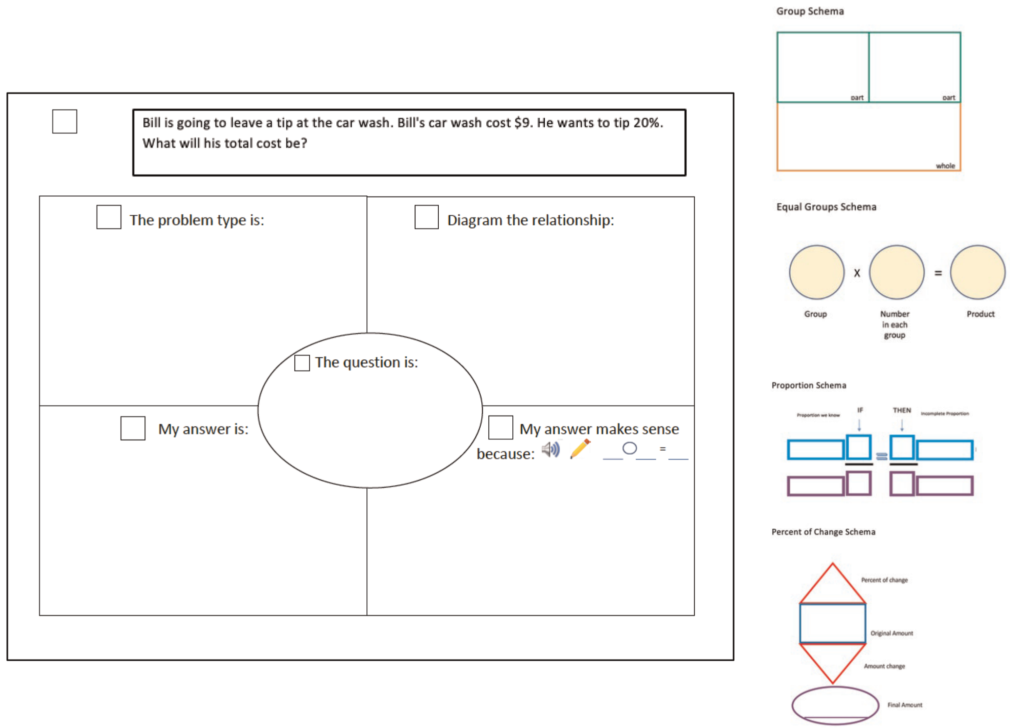

Figure 1. Heuristic and Schemas.

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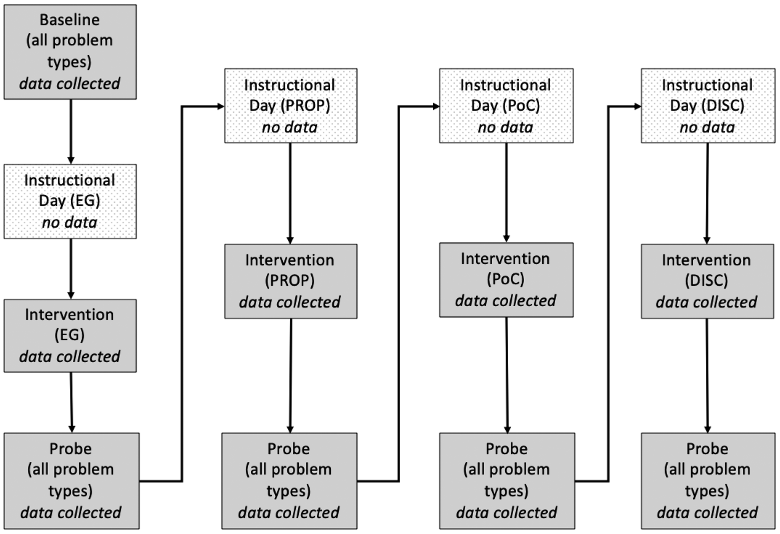

Note: Procedures in Baseline and Probe condition were identical.

Figure 2. Flow Chart of Procedures.

Dependent variables. The primary dependent variable was mathematical practice skills, measured by a seven-point researcher-created rubric. The participant could earn up to seven points in each problem: (1) identify the question, (2) indicate the problem type, (3) select the correct schema, (4) diagram the mathematical relationship, (5) correctly calculate the numerical solution, (6) correctly answer the question including a label, and (7) explain mathematical reasoning (verbal or written). The participant earned one point for each step completed independently correct. The participant had the opportunity to solve two problems of the targeted problem type, for a total of 14 possible points. The seven-point rubric was aligned with the steps of the heuristic (shown in Figure 1) and measured skills similar to previous MSBI research (e.g., Root, Cox, et al., 2018; Root, Henning, et al., 2018). The interventionist used the same rubric across all phases and conditions (baseline, probe, intervention, and generalization) to take data on the dependent variable.

Interobserver agreement. To ensure mathematical practice skills were being measured consistently according to the coding manual, a secondary coder calculated interobserver agreement (IOA) on the primary dependent variable. The first author trained an undergraduate research assistant in data collection and intervention procedures. Training sessions consisted of examples/non-examples, model sessions, permanent product review, and video example review. Once the secondary coder reached 90% reliability, she coded sessions in person and watched recorded sessions if needed. Agreement and fidelity data were collected for a minimum of 30% of the sessions across all conditions and problem types (baseline/probe = 69% equal group [EG], 69% proportion [PROP], 69% percent of change [PC]; intervention = 66% EG, 100% PROP, 66% PC; generalization = 75% EG, 75% PROP, 75% PC). For any phase that the secondary observer was not able to code a minimum of 30% of the sessions in person, video recordings were used to calculate agreement and compliance. Disagreement discussions were held weekly for any differences in coding.

Procedural fidelity. Procedural fidelity (PF; the amount to which the intervention was

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implemented as intended) was also calculated to increase confidence that MSBI was responsible for the observed change in student behavior. PF was coded during the same sessions as IOA using a checklist of interventionist behaviors, including: (a) use of the system of least prompts script, (b) reminding the student to use the materials to show how to solve the problem and explain their reasoning, and (c) appropriate timing and order of the prompting hierarchy for each step of problem solving.

Procedure

General procedure. The interventionist began each session by telling the student how many word problems he would need to solve for the day (two for intervention sessions and six for baseline, probe, and generalization sessions). Then she reminded him it was important to show his work and explain his reasoning so that his teachers could understand how he solved the problem and why he solved it that way. The student had access to the same materials during baseline, probe, and intervention sessions (see Figure 1 for heuristic and schemas), but a modified worksheet with reduced supports during generalization sessions. See Figure 2 for a flowchart of procedures. Specific praise was provided across all sessions for behaviors unrelated to the dependent variable. Per teacher direction, the researchers used the token economy that was in place for the student during his regular school day during all conditions. The participant could earn marbles for his attention to the problem and effort in completing the problem, but not for any behaviors associated with the task analysis. For example, the student could earn a marble for writing neatly, but not for writing the correct number. The participant exchanged marbles for 10-min of "free time" for every 10 marbles earned. He had a wide variety of leisure activities to choose from, his most frequent choice was video game time or playing catch outside.

Baseline and probe sessions. Procedures in baseline and probe sessions were identical. The difference between the two conditions was timing, in that baseline took place prior to intervention and probes took place after. The participant solved six problems (two of each type) during each baseline and probe session. The participant had access to all materials, but no instruction or feedback related to accuracy was provided. Problems were given one at a time in random order. When the participant was finished with a problem, he gave it to the interventionist and was given the next until all six had been completed.

Intervention sessions. The intervention procedures were developed following recommendations from previous research on MSBI (e.g., Cox & Root, 2020; Spooner et al., 2017) and followed the same sequence for each of the three targeted problem types. Intervention on each problem type began with a one-day initial lesson (e.g., instructional day in Figure 2) on the targeted problem type. This initial lesson included instruction on how to identify the type of problem (i.e., equal group, proportion, percent of change), academic vocabulary instruction needed for the problem type (i.e., sets, rate, equivalent), examples and non-examples of the problem type, how to represent the mathematical relationship in the appropriate schema (diagram; see Figure 1), and how to solve the problem using the heuristic that served as a task analysis (see Figure 1). The steps were to (1) read the problem aloud, (2) identify the question, (3) identify the problem type, (4) diagram the mathematical relationship (e.g., move the correct schema onto the heuristic and fill it in based on information from the problem), (5) solve for the unknown variable, and (6) explain mathematical reasoning. The interventionist then led the participant through several practice problems for each problem type using a model, lead, test format. No data were collected during this first lesson, as the participant did not have the opportunity to independently respond.

In subsequent intervention sessions for each problem type, the student solved two problems. The participant was given the instructional cue "show me how to solve this problem" and the opportunity to solve the problem independently. If the participant did

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not initiate a step within 30s, the interventionist used a system of least prompts. The prompting hierarchy included three levels: gesture (e.g., physically pointing to step of heuristic), specific verbal (e.g., saying "Step two is identify the problem type"), and model (e.g., "my turn, step two is identify the problem type, I know this is an equal group problem because we have ___ equal groups of _____, your turn, what type of problem is this?") An error correction procedure was used anytime the student incorrectly completed a step or skipped a step, which included a model with a re-test. Only independently correct steps are included on the graph (see Figure 3).

Generalization sessions. During generalization sessions the student was again reminded about the importance of showing his work and explaining his reasoning. To assess mathematical practices when provided with reduced supports, he did not have access to the schemas or the full heuristic during generalization probes. Generalization worksheets were printed on vertical computer paper with the word problem typed at the top and a check-list of the heuristic steps typed underneath the word problem. Just as in baseline/probe sessions, feedback on accuracy or performance related to the dependent variable was not given during generalization sessions.

Social Validity

The director of the private school and the student's teacher completed a researcher created questionnaire at the conclusion of the study. The questionnaire was designed to evaluate the school professionals' opinions on the usefulness of the content being taught as well as any observed behavioral changes (related to mathematics) for the specific student. Example questions include: (1) Do you feel it is important for students to explain their mathematical reasoning when solving word problems, why or why not? and (2) Did you observe any changes in Ron's academic performance during mathematics instruction? Unfortunately, due to the school year ending, the student was not available to answer social validity questions.

Results

Mathematical Practices

Results of the multiple-probe across behaviors design are displayed in Figure 3 Prior to intervention, Ron demonstrated a stable pattern of responses for each problem type, with the highest use of mathematical practices for EG problems (range = 0 – 3/14 points, mean = 1.5 points), then PROP (range = 0–1/14 points, mean = .67 points), and finally PC (range = 0-0/14 points, mean = 0). In the first EG intervention session, Ron's mathematical practice skills immediately increased to the score of 11/14 for EG problems, after which he reached ceiling scores (14) for two concurrent EG intervention sessions. In the first probe phase, Ron maintained his mathematical practice skill performance for EG problems, but improved slightly in his use of mathematical practice skills for both PROP (range 4–6/14, mean = 4.6 points) and PC problems (range = 2-2/14, mean = 2 points).

During the PROP intervention phase, Ron's mathematical practice skills during PROP problems immediately increased to 9/14 points, with an increasing trend over five sessions. He reached mastery criteria of at least 12/14 points (85% correct) over two sessions during his third and fifth intervention session. During the second probe phase, Ron maintained his mathematical practice skills for PROP problems (range = 9–13/14, mean = 11/14 points), but his mathematical practice skills for EG problems decreased (range = 6-6/14, mean = 6 points). This overgeneralization trend continued across all three problem types, where Ron solved all six problems using the procedures for the most recently taught problem type.

After MSBI instruction for PC problems, Ron's use of mathematical practice skills immediately increased from 2/14 points to 12/14 points when solving PC problems. He consistently earned 12/14 points over three PC intervention sessions, most often missing points for not explaining his reasoning accurately. During the third probe phase, Ron expressed a great deal of frustration when he tried to solve all six problems as though they were PC problems, and he could not find a

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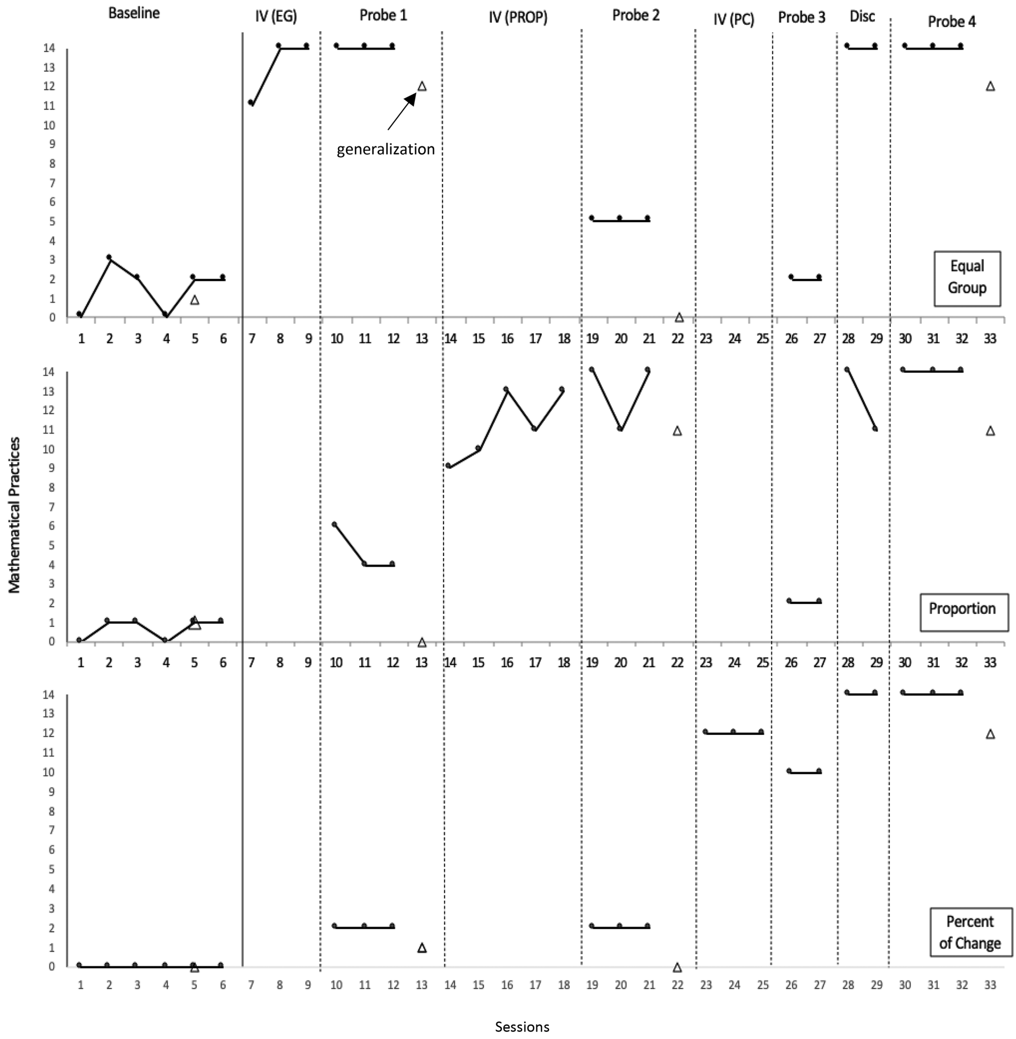

Figure 3. Graph.

way to complete the schematic diagram with the information provided. His frustration was apparent as he expressed "this doesn't make sense" and picked at his skin. The sessions also took double the amount of time (over 40 minutes). For these reasons, the decision was made to only conduct two sessions in the third probe phase, as it demonstrated a pattern of behavior but did not subject the student to a third highly frustrating session.

For the fourth intervention phase, the student was taught to discriminate between the three problem types using a T-chart and example problems in one 15 minute session. After the 15 minute discrimination session, data were collected for the final intervention

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sessions. During the discrimination intervention sessions, Ron's mathematical practice skills increased for all three problem types and reached ceiling level performance for EG and PC problems during both sessions. His mathematical practice skills for the PROP problems immediately increased to ceiling levels during the first discrimination intervention session, and demonstrated a minor drop to 11 points in the second discrimination intervention session. For the fourth and final probe phase, Ron exhibited full mastery of mathematical practice skills for all three problem types with three consecutive ceiling level scores (14/14). With three demonstrations of an effect between baseline and intervention scores, and maintaining those scores after a discrimination training phase, visual analysis confirms a functional relation between MSBI with discrimination training and mathematical practice skills.

Generalization

Ron's use of mathematical practice skills during generalization sessions followed a similar pattern to his use mathematical practices in probe sessions. During the baseline condition, Ron earned 1/14 points for mathematical practices when solving EG problems with reduced supports, 1/14 points when solving PROP problems, and 0/14 points when solving PC problems. Ron's use of mathematical practices during the baseline generalization session fell within his observed mathematical practice points for that condition. In the first probe phase, Ron's use of mathematical practices when solving problems with reduced supports increased for the EG problems (12/14 points), but remained consistent for the untaught problem types of PROP (0/14 points) and PC (1/14 points). During the second probe phase, Ron's use of mathematical practices with reduced supports decreased for EG problems (0/14 points), but increased for the recently taught PROP problems (11/14 points), and remained consistent for the untaught PC problems (0/14 points). Due to student frustration levels, no generalization probe was given during the third probe phase. In the final probe phase after discrimination training, Ron's use of mathematical practice points increased for all three problem types (EG = 12/14 points, PROP = 11/14 points, PC = 12/14 points). These final generalization scores were slightly lower than his performance for similar problem types when he had access to the full supports (14 points).

Fidelity and Reliability

The level of procedural fidelity (PF) remained high throughout the sessions, at 100% across all behaviors (i.e., problem types) during the baseline/probe condition and the generalization condition. In the intervention condition, PF was calculated at 100% for equal group problems, 97.2% for proportion problems (range = 93–100), and 93% for percent of change problems (range = 86–100). Interobserver agreement (IOA) also remained high across all conditions. During the baseline/probe condition, IOA was calculated as 98% for equal group problems (range = 92–100), 99.5% for proportion problems (range = 95–100), and 98.7% for percent of change problems 98.7% (range = 86–100). For both the intervention and generalization conditions, IOA was calculated at 100% across all problem types.

Social Validity

An important consideration for all researchers conducting intervention research is the social validity of the intervention for the student or society as a whole (Ledford & Gast, 2018). One aspect of social validity is the importance of the learned behavior for the student. In this study, the teacher and the school's director both felt mathematical reasoning and problem solving are important skills for all students at their school to be exposed to. During open ended questions, the director responded "I do believe that it is important for learners to talk through their mathematical reasoning while solving problems. I feel that having this skill allows learners to better communicate their ideas to staff and peers. In turn, this allows them to access help from others when solving problems that are above their ability level." A second aspect of social validity is the observation of a meaningful behavior change. Ron's use of

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mathematical practices clearly increased over his participation in the study. The teacher and director also indicated that Ron's participation in mathematics lessons improved during class instruction. The teacher said "We did see an increase in the amount of communication and reasoning Ron engaged in during math lessons. This is seen the most when he's explaining his work after solving a problem incorrectly or when he gets stuck while solving a difficult equation."

Discussion

Mathematics education emphasizes the importance of providing opportunities for students to engage in problem solving activities. Experts and state standards (e.g., NCTM, 2000; National Governor's Association, 2010) encourage teachers to engage students in mathematical practices to develop a deep understanding of mathematical content. Students with ASD often struggle with applied mathematical problem solving. As a result of deficits in executive functioning, metacognition, and communication, students with ASD will need social and academic supports to engage in mathematical practices. The purpose of this study was to evaluate the effects of MSBI on the use of mathematical practices of one student with ASD without co-morbid ID. The participant was taught to identify the question, analyze the problem structure, apply appropriate tools to model the mathematical relationship, solve for the problem solution, and explain his mathematical reasoning to support his answer. Using visual analysis, three demonstrations of an effect at three different points in time confirm a functional relation between MSBI and the student's mathematical practices. These findings provide evidence that MSBI may help students with ASD without co-morbid ID increase their use of mathematical practices when solving word problems.

Previous research (e.g., Root et al., 2017; Root, Cox, et al., 2018; Root, Henning, et al., 2018) has consistently demonstrated the effectiveness of MSBI to improve mathematical problem solving skills of students with ASD and co-morbid ID. Findings from this study extend existing literature by including a student with ASD without ID, and explicitly measuring mathematical practices. Ron was able to increase the number of mathematical practices employed across all three problem types, demonstrating an ability to conceptually understand the problem structure and employ appropriate strategies. The problem solving heuristic, systematic prompting, visual supports, and explicit instruction used in MSBI not only taught Ron to solve the word problems, but also taught Ron to monitor and communicate his mathematical thinking.

This study also extends previous findings by examining the possibility of fading visual supports. Two students with ASD without co-morbid ID demonstrated possible reliance on the visual supports in a 2020 study by Cox and Root. The findings in this study demonstrate that some students with ASD can maintain use of mathematical practices after visual supports are faded. It is possible the more generalized problem solving heuristic used in this study facilitated Ron's use of use of mathematical practice skills when supports were reduced during generalization sessions, by minimizing the amount of support provided during intervention. These findings highlight the importance of considering the level of supports needed for individual students.

The majority of participants in previous MSBI research have required explicit discrimination training to differentiate between problem types (e.g., Root, Cox, et al., 2018; Root et al., 2019; Root, Henning, et al. 2018). Teaching each problem type to mastery, followed by measuring problem solving skills for all three problem types, allowed for observation of over-generalization patterns of behavior demonstrated by the student. Consistent with findings from other studies (e.g., Root, Cox, et al., 2018; Root, Henning, et al., 2018), Ron also over-generalized problem solving strategies to novel problem types, and needed additional instruction to discriminate between problem types. What is still unknown is which students are most likely to benefit from explicit training beyond traditional MSBI.

Interestingly, although Ron required explicit discrimination training to differentiate between problem types, he did increase the use of some mathematical practice skills after the first intervention to untaught

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problem types. These findings suggest these skills are not completely behaviorally independent. This is an important finding for both researchers and teachers. Researchers should consider utilizing designs that stagger the independent variable across more independent tiers rather than behaviors (such as multiple probe across participants). Teachers can benefit from this information, recognizing that explicitly teaching problem solving skills in one problem type may promote those skills in different problem types, but will likely not lead to mastery of different problem types without additional instruction.

Implications for Practice

Teachers can employ schema-based instruction with various levels of modification to support a variety of learners with ASD as they learn how to solve both additive and multiplicative word problems. Modifications such as providing a visual diagram can help students by reducing fine motor requirements of drawing the schemas independently. Additionally, supplying the diagrams provides a stimulus prompt for students where they can write in a defined space, helping them to keep their representations organized and well defined. A task analysis or a heuristic with more generic steps can help students monitor their progress in a visual way. These visual supports may be more beneficial than a mnemonic for students with ASD, and this study provides evidence that these visual supports can successfully be faded over time. Additionally, using systematic feedback such as a system of least prompts can help students with ASD adjust their behaviors and experience a higher rate of success in a quicker time period. Finally, it is clear that some students with ASD will need additional instruction to help them discriminate between problem types. Therefore, teachers should monitor student progress when teaching a new problem type to ensure that students are maintaining performance for previously taught problem types without over-generalization. If over-generalization does occur, teachers can use explicit instruction in the form of a model-lead-test procedure with a T-chart to help students discriminate between problem types and maintain performance for previously and newly taught problem types. The ability to discriminate between problem types is a critical skill for students to be able to apply these skills to novel problems and everyday applications.

Limitations and Future Research

This study provides evidence that MSBI is an effective instructional strategy to improve mathematical practices for a student with ASD, with evidence that visual supports can be faded after instruction. However, it is unknown whether these findings will generalize to other participants, or if they can be replicated when implemented in a more natural context with teachers as interventionists or in a whole-group setting. Systematic replication research is needed to add to the external validity, adding valuable information about what works for which students under what conditions. Additionally, because the participant's mathematical practice skills improved for all problem types after the first intervention phase for equal group problems, there is a threat to internal validity when measuring across behaviors. Ron's performance did stabilize and hit a ceiling during the first probe phase, but future researchers should consider alternative designs to maintain more rigorous experimental control. Designs such as multiple probe across participants would allow researchers to systematically implement intervention sessions across more independent tiers (i.e., participants).

Ron's director and teacher both felt the intervention was relevant and impactful on his mathematics performance. The student was unavailable to participate in social validity questions for this study, but future research should investigate how participants view the intervention in terms of enjoyment and usefulness. As the beneficiary of the intervention, it is important to consider the student's plan to continue using features of interventions, their enjoyment, and their recommendations for improving instruction in the future. This is a limitation of this study and should be addressed in the future.

One additional limitation of this study was that although the student needed

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discrimination training, it is unclear why he was unable to discriminate between the problem types. In fact, the student indicated that he knew the difference between the problem types, but thought he had to solve the problems the way the interventionist had most recently taught. In order to adhere to research protocol, the interventionist did not remind the student that he could use any of the schemas while he was solving the problems during the second probe phase, because he had not been provided that instruction previously. Therefore, it is possible that a simple prompt to use the appropriate schema could have reduced the student's stress level during the third probe and eliminated the need for discrimination training. However, it is also possible that the student only believed that he did not need the discrimination training, as students with ASD often have a difficult time self-evaluating and self-monitoring (Schaefer Whitby, 2012). Future research should design procedures to explicitly instruct the student to solve the problem using the appropriate strategy and/or schema to assess the need for discrimination training clearer.

Conclusion

The use of mathematical practices are especially important for students with ASD who are expected to make adequate yearly progress with the Common Core State Standards, which expect students to be able communicate their mathematical thinking, use representations and models to represent their mathematical reasoning, and monitor their problem solving process. This study contributes to the growing body of research supporting MSBI as an effective instructional approach to not only improve mathematical problem solving skills, but also to support students with ASD in these mathematical practices.

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